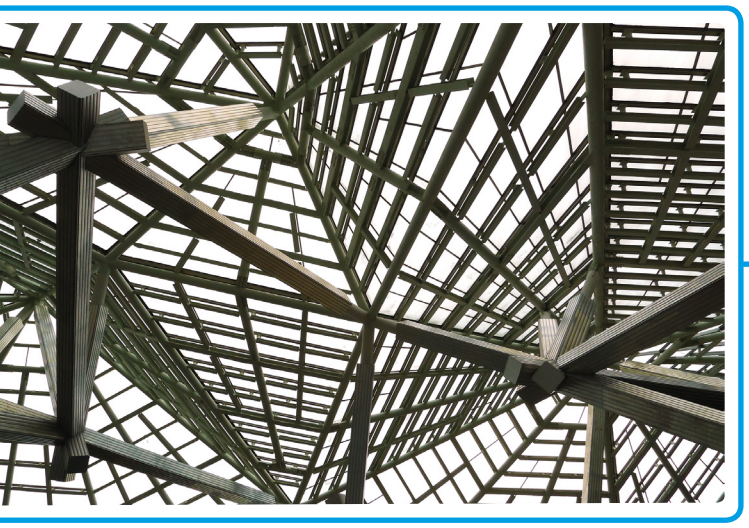
Diversification within an equity factor-based framework



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EXECUTIVE SUMMARY

- An understanding of the design choices underlying multi-factor products is crucial if investors are to avoid outcomes that may ultimately disappoint them.
- These design choices include: factor selection, starting universe, multi-factor construction approach, stock weighting scheme, factor weights, regional allocation and currency exposure.
- Using evidence and beliefs, we outline a "blank-sheet-of-paper" approach to designing a particular strategy that places a heavy emphasis on diversification at the factor, region, sector and stock level.
- This leads to considered objectives for portfolio return, risk and diversification which can be clearly messaged to investors.

As factor-based investing has increased in popularity since the financial crisis, so has the number of products available for investors to choose from. Underlying each of these products is a set of design choices whether they are explicitly or implicitly made. For investors we believe it is critically important that they understand these design choices in order to assess how a strategy is likely to perform in the different environments it will invariably face.

When we commenced with the creation of a multi-factor equity strategy, we took a "blank-sheet-of-paper" approach to the following design choices which were explicitly considered and incorporated within the end strategy:

- Factor selection
- Starting universe
- Multi-factor construction approach
- Stock weighting scheme
- Factor weights
- Regional weights and currency exposure

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Having explicit consideration of these areas facilitates clear messaging to investors with respect to the relevant objectives for the strategy and its characteristics. Underlying our strategy's philosophy is the Asset Allocation team's belief in the power of diversification which can be shown to not only reduce risk but to improve geometric returns[1]. Diversification can occur at different levels and is pivotal to the strategy's construction.

Below we highlight the design choices that we made in designing the strategy and elaborate on the evidence and beliefs which underpin these choices.

FACTOR SELECTION

LGIM's Asset Allocation team through its research and investment experience has developed beliefs on the merits of different factors. While a vast number of factors are documented in academic literature with over 300 found in one study[2], there are relatively few that have an established body of academic research associated with them. Those that we find to be covered more consistently include: value, low volatility, quality, momentum and size.

These correspond closely to the ERI Scientific Beta range of factors available[3]. The key distinction to be made is with regard to the quality factor. ERI Scientific Beta considers 'quality' to be composed of two separate and distinct factors, namely high profitability and low investment, which is in line with Fama and French (2014). We agree with this assessment though we are cautious in giving these two factors as much weight as more established factors. High profitability and low investment have only been published in the academic literature in this century while factors such as value, momentum, size and low volatility all have papers associated with them from the previous century. As such, our confidence in high profitability and low investment is reflected through an adjustment such that each receives half-weight. In essence, these two factors equally-weighted combine to form a single 'quality' factor.

Additionally, the Asset Allocation team has a prior belief that cross-sectional momentum (i.e. momentum at the stock level) is difficult to capture through regularly-rebalanced indices and may induce additional turnover without significant additional benefit, particularly within a multi-factor framework. Published papers by Koraczyk and Sadka (2004) and more recently Novy-Marx and Velikov (2016) support the belief of limited capacity for momentum strategies prior to alpha erosion though a working paper by Frazzini, Israel and Moskowitz (2015) challenges this wisdom. However, this latter paper uses proprietary data which cannot be scrutinised. Where momentum is used in portfolios, our Asset Allocation team's general preference is to target time-series momentum involving futures contracts rather than cross-sectional momentum involving individual stocks with the aim of reducing the transaction costs of trading momentum (and indeed Pedersen, Moskowitz and Ooi (2012) present evidence of time-series momentum's ability to completely explain cross-sectional momentum in equities). In our testing we retained momentum as a possible factor for consideration though it faced a higher hurdle for inclusion based on the prior belief.

As we will note later, the Scientific Beta High Factor Intensity (HFI) indices, which we have chosen to use, incorporate a filter which removes stocks with poor multi-factor scores. Momentum is an input into the multi-factor score which means that stocks that score poorly on momentum, all else equal, are more likely to be filtered out. We feel that by removing stocks with poor momentum rather than focusing on stocks with good momentum, this enables us to incorporate the factor in an efficient way.

Our aim in factor selection is to have enough factors such that factor diversification is effective though, crucially, we must have a high level of belief in these factors.

STARTING UNIVERSE

Given our aim is to construct a global multi-factor equity strategy, the key question with respect to the starting universe is whether to include emerging markets or restrict the choice to developed markets where there is already a significant body of research on the existence of factors. We find evidence of all the main factors above working in emerging markets as listed in the table on the next page[4] and as such we include this region within our universe. This improves our ability to diversify across the markets of more countries, many of which are less co-integrated with developed markets.

1 Humble and Southall (2014)
2 Harvey *et al* (2016)
3 ERI Scientific Beta is a provider of factor-based index strategies
4 Adapted from Shirbini (2016)

Figure 1: Factor research studies in Emerging Markets

Factor (Long/Short)	Sample	Period	Premium	Source
Value	Emerging Markets	1990-2011	1.15% (Monthly Mean)	Cakici, Fabozzi and Tan (2013)
Momentum	Emerging Markets	1990-2011	0.86% (Monthly Mean)	Cakici, Fabozzi and Tan (2013)
Size	Emerging Markets	1990-2011	0.28% (Monthly Mean)	Cakici, Fabozzi and Tan (2013)
Low Volatility	Emerging Markets	1999-2012	2.10% (Annual Mean)	Blitz, Pang and Van Vliet (2013)
Low Investment	Emerging and Developed Markets	1982-2010	6.18% (Annual Mean)	Watanabe, Xu, Yao and Yu (2013)
High Profitability	Emerging Markets (Europe)	2002-2014	0.71% (Monthly Mean)	Zaremba (2014)

MULTI-FACTOR CONSTRUCTION APPROACH

A key debate going on within factor investing circles surrounds the issue of multi-factor portfolio construction: whether to go "top-down" or "bottom-up" in constructing your factor exposure[5]. The "top-down" approach allocates to factors as individual building blocks. For example, a top-down multi-factor strategy might have allocations to a value portfolio, a quality portfolio and a low volatility portfolio (where each of these portfolios contains stocks that score strongly on their respective characteristics).

In contrast, the bottom-up approach to multi-factor investing gives each stock in the universe a score on each of the desired factors. These individual factor scores are then combined into an overall multi-factor score for each security in the universe. This composite score is then used to derive a weight in the multi-factor portfolio. There is variation in methodology across different bottom-up strategies.

Additionally, ERI Scientific Beta in 2017 introduced its new range of "High Factor Intensity" (HFI) multi-factor indices which retain the overall top-down structure used within its original "Multi-Beta Multi-Strategy" range of multi-factor indices though it adds a bottom-up filtering process applied within each factor sleeve as described in Amenc *et al* (2017). We see this as a way of effectively synthesising the top-down and bottom-up approaches. It preserves the simplicity and transparency of the top-down approach but accounts for the cross-factor interaction which, until now, has only been captured by bottom-up approaches. The key difference from other providers we find is in its use of the bottom-up part of the process. Here ERI Scientific Beta focus on eliminating stocks with poor multi-factor scores rather than adding weight to stocks with strong multi-factor scores – a method which is prevalent amongst most pure bottom-up approaches.

In order to understand the difference between the bottom-up approach, the top-down approach and the two step-filtering approach present in the HFI methodology (from here on referred to as the "combined" approach), we conducted our own independent empirical research to understand strategy characteristics. One of the key challenges in comparing approaches across providers is due to differences in factor definitions, stock weighting schemes or stock universes, amongst others. Hence it was important to construct strategies using a uniform set of inputs apart from the multi-factor construction approach in order to create a true "apples-to-apples" comparison.

Our dataset included stock-level factor information for a global universe of stocks (including developed and emerging market stocks) between March 2002 and December 2016. The four factors we included were value (book-to-price), low volatility (based on 1 year daily returns), momentum (last 12 month return omitting the most recent month) and quality (equal weight between high profitability – gross profits-to-assets definition -

5 See Fitzgibbons *et al.* (2016) and Bender and Wang (2016)

and low investment – based on three year asset growth). These four factors were given equal weight. We formed portfolios that were semi-annually rebalanced at the end of February and the end of August. Two stock weighting schemes were used for all multi-factor construction approaches: capitalisation-weighting and equal-weighting.

We then tested the various multi-factor construction approaches including: (i) a top-down approach based on top x% selection within each factor sleeve where x = 15, 30 and 50, (ii) three different bottom-up approaches including geometric S-score multi-factor scores, arithmetic S-score multi-factor scores and average factor rank multi-factor scores[6], all based on top x% selection for the overall portfolio where x = 15, 30 and 50 and (iii) the combined approach which uses a top 50% initial selection per the top-down approach in (i) followed by a top 60% selection based on average factor rank multi-factor scores within each factor sleeve. Additionally, we examined portfolios incorporating region-neutrality, sector-neutrality as well as region and sector-neutrality alongside the unconstrained version. This led to 104 different multi-factor portfolios being formed (2 weighting schemes x 13 multi-factor construction approaches x 4 portfolio constraint options). We note though that our list of approaches tested is far from exhaustive and indeed only scratches the surface with some of the most common found within the industry.

When examining the results, two measures of risk-adjusted return we consider are beta-adjusted information ratio (i.e. beta-adjusted active return over beta-adjusted tracking error[7]) and Sharpe ratio. We found that the combined approach which features in the HFI methodology stacked up well against the various other methodologies. The combined approach with region-neutral formation and equal-weighting, which maps closest to the methodology in HFI indices, had a beta-adjusted information ratio in the top decile and a Sharpe ratio in the 3rd decile. However we would de-emphasise the importance of this empirical testing. We saw the testing as validating our belief that the combined approach is an efficient implementation rather than it driving our decision.

Figure 2: Multi-factor construction approach

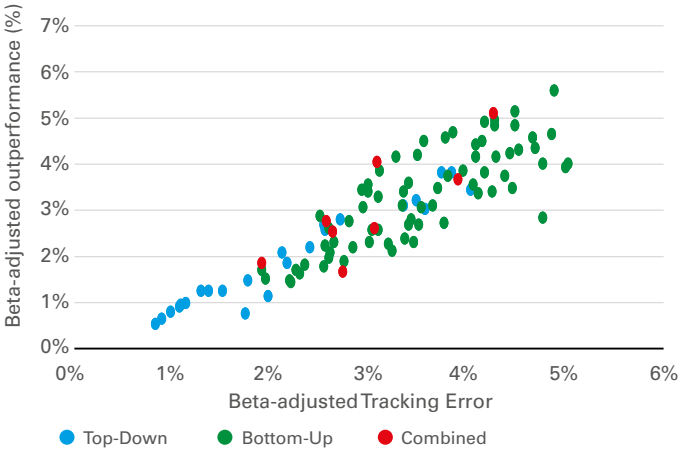


Figure 3: Narrow vs. broad stock selection

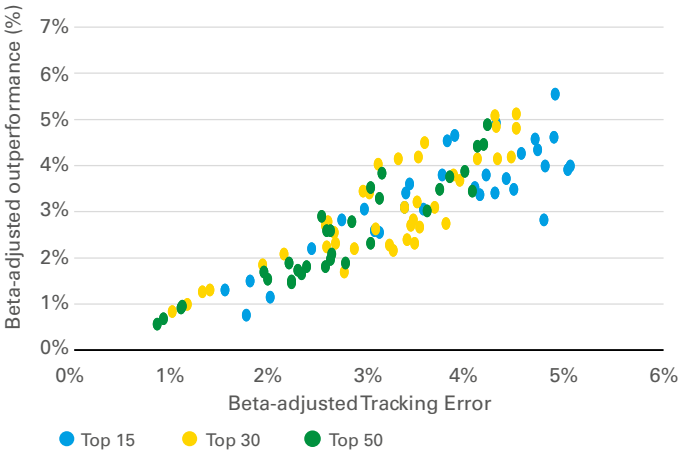


Figure 4: Stock weighting scheme

Source: LGIM

6 See appendix for details on the formulas for these three variants of bottom-up multi-factor scores

7 Beta-adjusted active return is what is known otherwise as "Jensen's alpha" or "ex-post alpha" as described in Jensen (1967) using the Capital Asset Pricing Model (CAPM) beta to adjust active returns. Beta-adjusted tracking error is a related statistic equal to the volatility of the beta-adjusted active returns. We prefer the beta-adjusted information ratio over the standard information ratio as it does not automatically penalise strategies with beta less than 1 (which is a desirable characteristic for some investors).

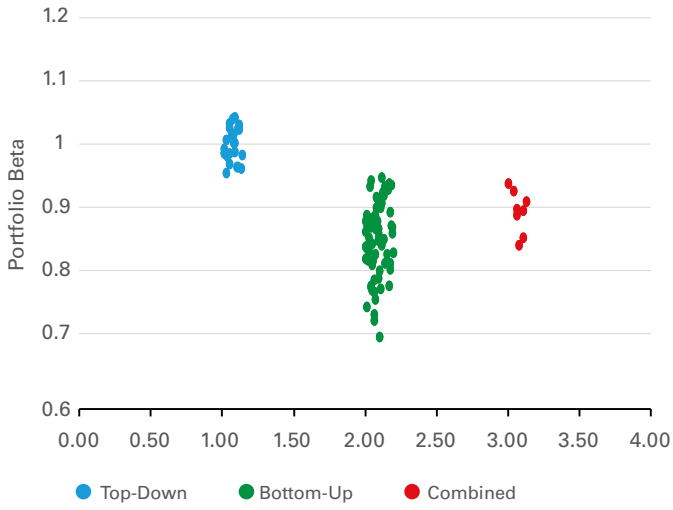
Bottom-up strategies tended to carry higher beta-adjusted tracking errors which were generally commensurate with higher beta-adjusted returns though this link weakened for concentrated approaches (i.e. top 15% selection)[8]. Overall on this risk-adjusted return measure we found that there was generally a linear relationship between risk and return for all strategies except those with particularly high beta-adjusted tracking errors which mostly corresponded with concentrated portfolios.

Also, we noticed that bottom-up strategies tended to carry a low beta bias over the time period. This finding is similar to that of Jivraj *et al*. (2016) who also look at multi-factor construction approaches that include the low volatility factor for a US universe between January 2003 and July 2016.

In terms of stock weighting schemes, we find that equal-weighted strategies dominated cap-weighted strategies with this being robust to examining time periods when the size factor produced a zero return. This would indicate that the performance of the size factor may not have been the only driver of the performance differential but could be due to the effects of better diversification and lower stock-specific risk for equal-weighted strategies.

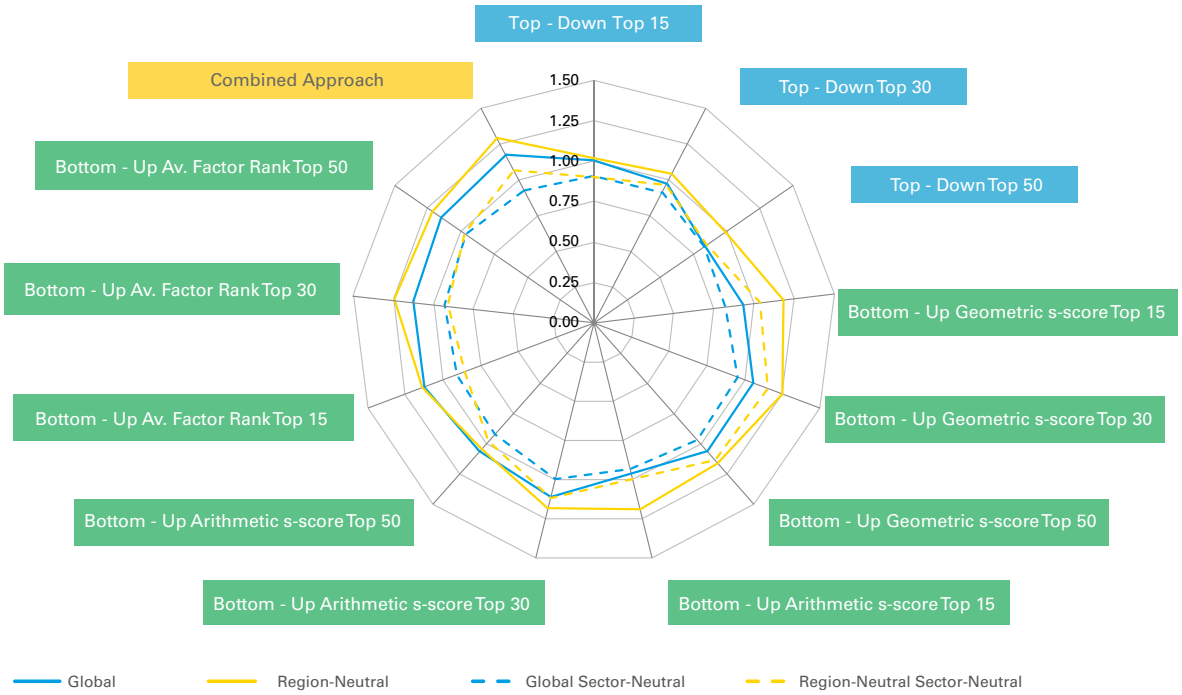
Additionally, we found that while region-neutrality (formed using 11 regional building blocks akin to the approach taken by ERI Scientific Beta in its methodology) led typically to improvements in beta-adjusted information ratio relative to a global approach to stock selection, this also typically led to small declines in Sharpe ratio. We found little support for either sector-neutrality or region-and-sector-neutrality on a performance basis where sector-neutrality was achieved through a re-scaling process back to market cap sector weights (at the global level for sector-neutral and within region for region-and-sector neutral). We also note the higher turnover of these strategies, particularly for region and sector neutrality.

Figure 5: CAPM beta for different multi-factor construction approaches



Source: LGIM

Figure 6: Beta Adjusted Information Ratio: Equally-Weighted Portfolios



Source: LGIM. For illustrative purposes only

8 Note that the combined approach leads to the equivalent of a top 30% selection (i.e. a top 50% selection followed by a top 60% selection)

While we do not believe that the time period used in our research is sufficiently long to make definitive conclusions, we would argue that it still provides some level of insight. We acknowledge the results in Amenc *et al* who use US stock data over the period 1975 to 2015 to confirm the robustness of the Scientific Beta HFI approach relative to a concentrated bottom-up approach. Similarly, Leippold and Rueegg (2017) who use US stock data from 1963 onward, find a similar pattern as us with regard to the low beta bias of bottom-up strategies that include the low volatility factor while also finding similar levels of risk-adjusted return between top-down and bottom-up approaches.

Overall, having undertaken the independent research above, our results seemed to favour the combined approach which aligns with the methodology within ERI Scientific Beta HFI Indices. This validated our belief that the combined approach is an efficient way of integrating bottom-up and top-down approaches. As such, we decided to use indices within this range to implement our multi-factor strategy.

STOCK WEIGHTING SCHEME

When considering how to weight individual stocks after the stock selection process, our goal is to seek diversification such that stock-specific risk is reduced. Our Asset Allocation team, when deciding what weight to give assets, look at both capital weights as well as risk weights. Diversifying by capital weights corresponds to the ERI Scientific Beta Maximum Deconcentration stock weighting scheme while diversifying by risk weights corresponds to the ERI Scientific Beta Diversified Risk Weighted stock weighting scheme. As such we use an equal-weighted combination of these two weighting schemes. This leads to a significant reduction in stock-level concentration relative to cap-weighted indices[9] which is a common aim for many investors.

By seeking this diversified stock weighting scheme, we note that we also by proxy achieve more diversified sector weights. This has the effect of reducing the influence of the largest sectors which could be susceptible to periods of over-valuation[10].

FACTOR WEIGHTS

Our primary objective when setting factor weights is to seek diversified factor exposure. This means that we want to ensure that we are carrying significantly positive and relatively balanced exposures to the factors we are targeting through the economic cycle. Our starting point was to test equal factor weights and if then there were a need to deviate from this position we would have done so. However, as the ERI Scientific Beta HFI methodology explicitly accounts for cross-factor interactions, we expect factor balance to naturally occur.

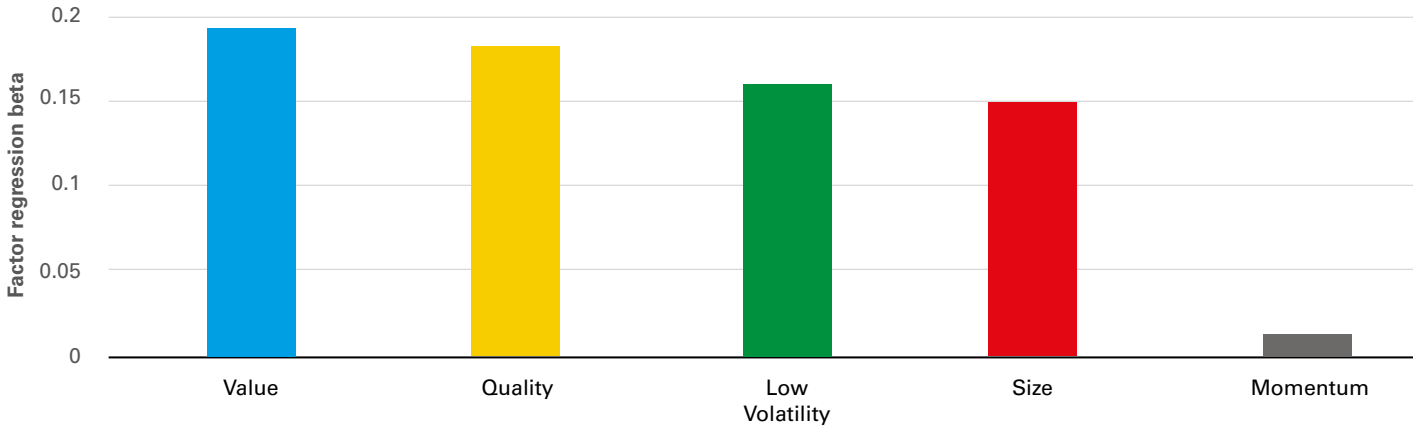
Additionally, as we opted for a diversified stock weighting scheme, we recognised that this introduced a significant amount of size factor exposure itself and an explicit allocation to the factor would lead to an imbalance that would go against our objective. We tested different factor weights over two time periods: for a US stock universe from 1975 to 2015 and for a global stock universe from 2002 to 2016. We saw broadly similar overall return and risk statistics across different combinations of factors including those that held momentum and size factors though critically there were differences when it came to factor exposures.

These factor exposures were defined by the factor regression beta from a seven factor regression that included the market factor alongside six long/short factor portfolios (value, high profitability, low investment, low volatility, size and momentum). We noticed a heavy imbalance in factor exposures when including the Size factor which matched our initial intuition. When looking at the momentum factor exposure over the US long-term backtest, while still being statistically significant, it was markedly lower in magnitude relative to the other factors (considering high profitability and low investment as a single factor). This meant that possibly momentum had indeed been more difficult to capture over the long-term. This gave us sufficient reason to exclude momentum as we had not seen much evidence to challenge our prior belief.

9 As an indication of the reduction, the percentage weight in the twenty largest stocks in our strategy in June 2017 was about 5% while this was about 15% for a representative global market-cap equity index

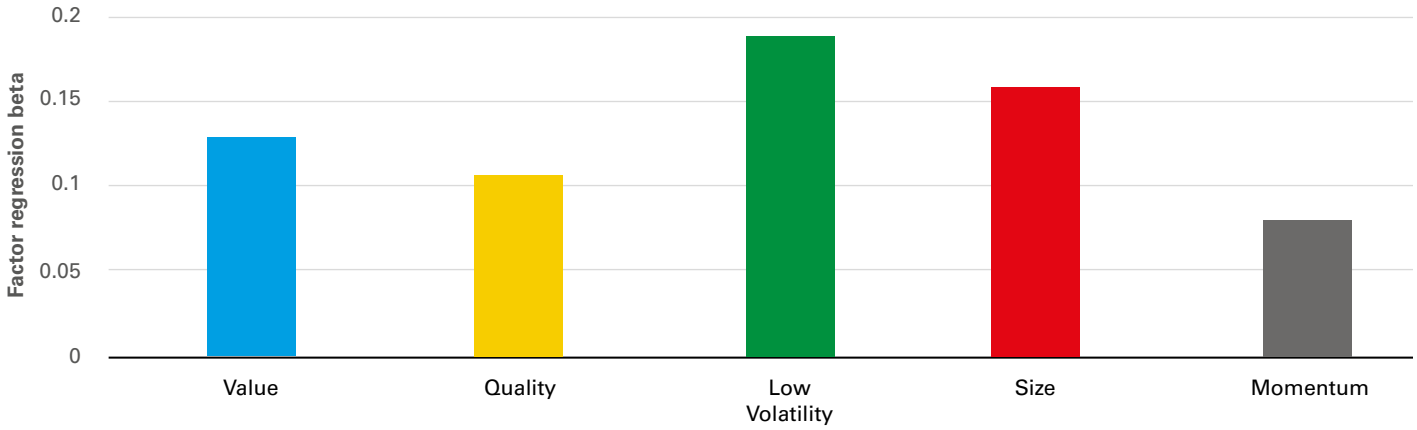
10 For example, the Information Technology sector during the "dot-com" bubble came to represent about 30% of the S&P 500 at one point

Figure 7: Factor exposures in US backtest (1975 – 2015)



Source: ERI Scientific Beta

Figure 8: Factor exposures in Global backtest (2002 – 2016)



Source: ERI Scientific Beta

As a result of excluding explicit allocations to momentum and size, we were left with equal weighting value, low volatility and quality factors (where quality itself was an equal-weighted combination of high profitability and low investment). We note that we achieve very good factor balance across the factors targeted in the US long-term backtest. The balance is not as good in the shorter-term global backtest, though we felt this was still quite reasonable. All factor exposures across the two backtests are significant at the 1% level except for momentum in the US long-term backtest. While momentum is positive and significant in the shorter-term global backtest (even though it is not explicitly targeted), we would not expect this to be the case over all time periods given the result of the longer-term US test.

This leaves us with diversified factor exposure, which added to diversification at stock and sector level, all enable us to reduce the risks of particular factors, stocks or sectors performing poorly.

REGIONAL WEIGHTS AND CURRENCY EXPOSURE

The final layer of diversification we seek is with regard to the strategy's regional weights and currency exposures. Often a multi-factor strategy's regional weights are an artefact of the stock selection process. We believe this risks the introduction of unintended regional bets. An explicit regional allocation process can alleviate the issue.

When deciding upon regional allocations, the LGIM Asset Allocation team splits the global equity universe into six distinct regions: United Kingdom, developed Europe ex United Kingdom, North America, Japan, developed Asia-Pacific ex Japan and Emerging Markets[11].

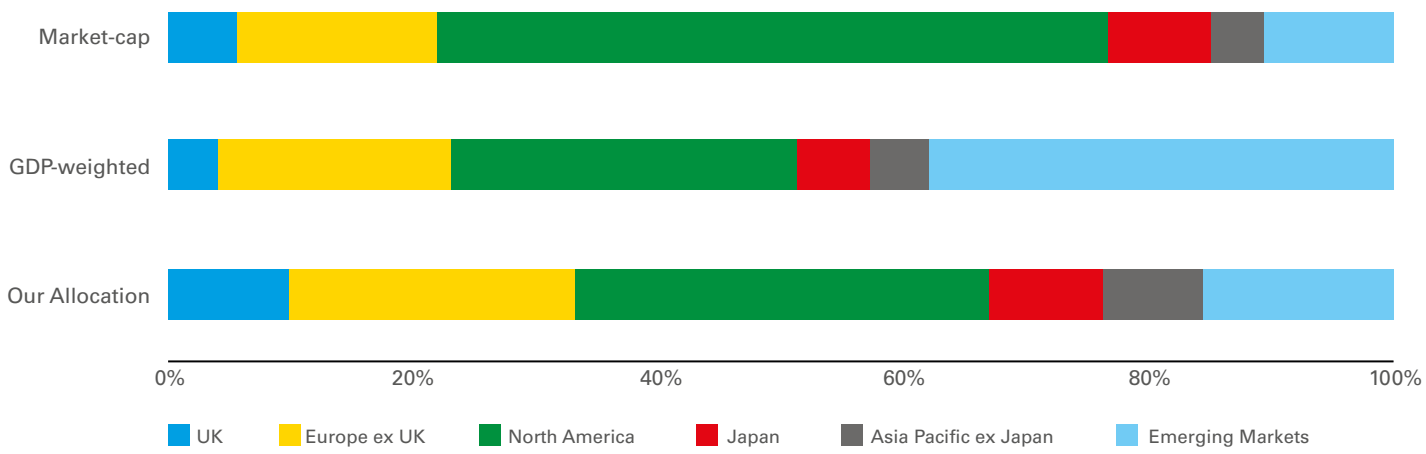
Individual regional weights can be chosen in many alternative ways. In addition to an equally weighted allocation, two of the most intuitive alternatives include market-cap weighting and GDP-weighting.

The market cap-weighting approach is widely adopted in index-based investing due to its straightforward implementation, removing the need to rebalance amongst regions[12]. However, we recognise that one of the key aims of investors when considering investments in this area is to avoid links to market-cap weighting as it can re-introduce the sensitivity to company valuations. As valuations of an individual stock or a group of stocks within a region increase, this would drive the weight of the region where these stocks are listed higher. Furthermore, such an approach results in a large concentration in North America, with nearly 60% weight in the region at present.

Nevertheless, while acknowledging its limitations, the market-cap weights of individual regions represent their importance in financial markets and as such they remain a dimension worthy of consideration. However, we believe it should be accounted for in conjunction with the regions' economic significance that is reflected in their gross domestic product (GDP). The GDP-weighted approach breaks the link between country weightings and market-cap size, hence reducing the sensitivity of regional exposures to changes in market sentiment. Consequently, the weight of larger emerging market economies like China will be higher and the overall regional exposure could be significantly different from the conventional market-cap benchmark.

The regional allocation we chose for our global multi-factor equity strategy aims to reflect both the economic and the financial significance of individual regions to provide a more diversified exposure that is not overly reliant on any single region. To enhance that diversification even further we will marginally increase the weight of those regions that are less correlated with the home market. For a UK investor, this would normally mean a positive adjustment to Asia-Pacific, including Japan, and emerging markets on a stand-alone basis. For investors based elsewhere, for example in the Eurozone, these adjustments would be different. We would ensure that the chosen allocation does not result in a concentrated exposure to the politics of a specific country. That would lead to a reduction in weight of regions such as United Kingdom and Japan, and to a slightly lesser degree, North America. Finally we would also consider governance standards to determine whether investors get the returns they earn for taking the equity risk in a particular region. Our approach results in a more-balanced regional allocation which further reduces the strategy's concentration risk.

Figure 9: Regional weights



Source: LGIM

11 Note that this is similar though different to the 11 ERI Scientific Beta regions (8 developed, 3 emerging) that were used in our multi-factor construction approach research detailed previously

12 Although rebalancing would still occur within regions as factor data changes

Interestingly we noted that by lowering the weight of the North American region we were able to increase stock-level diversification as measured by the effective number of stocks[13]. Our multifactor strategy using market cap regional weights resulted in an effective number of stocks figure of 657 while the same strategy using our regional weights had a figure of 852 as of December 2016. This increase was possible due to the high average stock weight for the North American universe relative to other regions. By moving weight to other regions, this led to greater diversification at the stock level as measured by effective number of stocks.

In addition to the regional allocation, for our UK-based strategy, we hold currency exposures which hedge 50% of the overseas (i.e. non-GBP) developed markets currency risk within the strategy. We believe this reduces volatility over the long-term while not sacrificing return, see Joiner and Mollan (2017).

There is an ongoing monitoring process in place for the strategy with our Asset Allocation team having discretion to change elements of the strategy in order to be able to continue to deliver on its objectives. Nevertheless, we expect any changes, including the regional allocation, to be gradually made over time reflecting the strategic rather than tactical nature of the process.

CONCLUSION

We have demonstrated above the evidence and beliefs that underpin the design choices we made for our multi-factor strategy. Choices were made with regard to the selection of factors, the starting universe, the multi-factor construction approach, factor weights and the stock weighting scheme as well as the regional allocation and currency exposures. Explicit consideration was given to many different elements that can influence outcomes.

This level of understanding also allows us to create well-informed objectives for return and risk that can be messaged to investors. For our strategy, based on its exposure to the targeted factors, we aim to, over the long-term, outperform a blend of market-cap indices with a similar regional weighting to ours, at a lower level of volatility.

By seeking diversification at multiple layers including at factor, region, sector and stock level, we have designed a solution which we believe will meet the objectives of many investors who are looking for a strategic, long-only exposure to equity factors delivered in a diversified manner.

13 The effective number of stocks is defined as the reciprocal of the Herfindahl Index, which is a commonly used measure of portfolio concentration:

$$\text{Effective number of stocks} = \frac{1}{\sum_{i=1}^{N} w_i^2}$$

where N is the number of constituent stocks in the index and w_i is the weight of stock i in the index. In brief, the effective number of stocks in a portfolio indicates how many stocks would be needed in an equal-weighted portfolio to obtain the same level of concentration (as measured by the Herfindahl Index). Equal-weighting stocks in a portfolio will lead to the maximum effective number of stocks.

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APPENDIX

Geometric S-Score and Arithmetic S-score

In order to calculate S-Scores for stocks, first we calculate z-scores.

$F_{j,i}$ is stock i's attribute value for factor j

μ_j is the cross-sectional mean (i.e. the average across all stocks) for factor j

σ_j is the cross-sectional standard deviation (i.e. the average across all stocks) for factor j

$z_{j,i}$ is stock i's z-score for factor j

$$z_{j,i} = \frac{(Fj,i - \mu j)}{\sigma_j}$$

We then apply a winsoring process to the z-scores such that values above 3 are set to 3 and that values below -3 are set to -3. The z-score formula above is re-run with the new values and the winsoring process is applied repeatedly until all z-scores in the universe fall between -3 and 3. We use these winsorised z-scores to calculate the S-scores.

$S_{j,i}$ is stock i's S-score for factor j

$$S_{j,i} = \int_{-\infty}^{z_{j,i}} \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

The z-score is mapped to an S-Score using the cumulative distribution function of the standard normal such that it lies between 0 and 1.

For stock i its geometric S-score (GS) multi-factor score (MFS) across k factors is:

$$GS\ MFS_i = \prod_{j=1}^{k} S_{j,i}$$

And its arithmetic S-score (AS) multi-factor score across k factors is:

$$AS\ MFS_i = \frac{1}{k}\sum_{j=1}^{k} S_{j,i}$$

Average Factor Rank

For a universe with n stocks, the attribute rank for stock i on factor j is defined by:

$$R_{j,i} = \frac{Rank\ (F_{j,i})}{n}$$

Then the average factor rank (AFR) multi-factor score for stock i with k factors is simply given by:

$$AFR\ MFS_i = \frac{1}{k}\sum_{j=1}^{k} R_{j,i}$$

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